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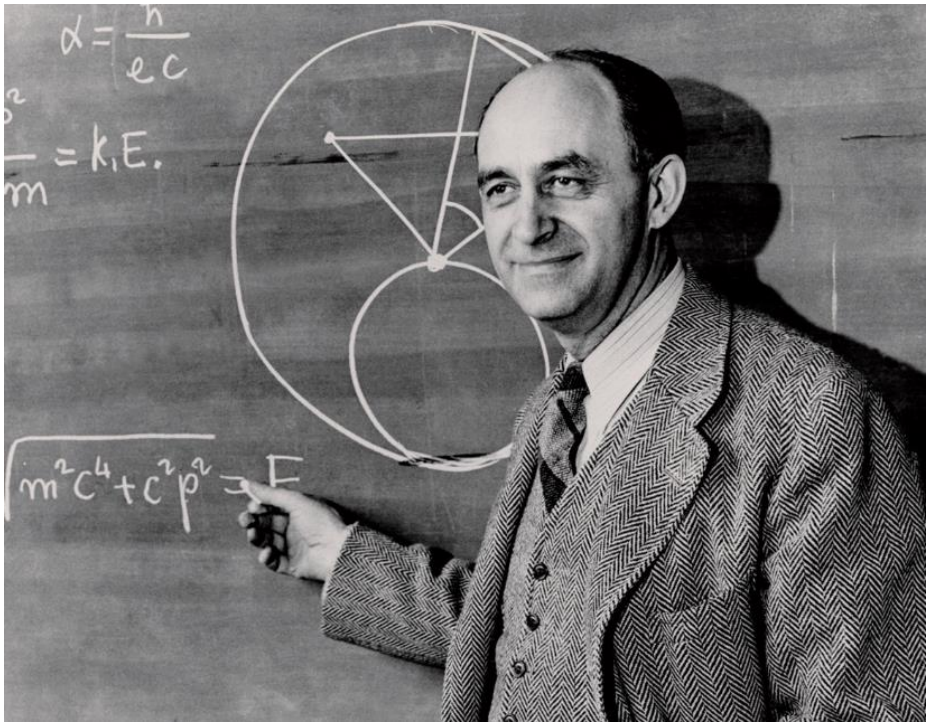
REDHILL ACADEMY TRUST

Key Stage 3
Maths Enrichment Week
29th June to 3rd July

Fermi Estimation Challenge

Enrico Fermi



Enrico Fermi was an Italian American physicist and the creator of the world's first nuclear reactor.

Fermi was known for his **ability to make good approximate calculations with little or no actual data**.

He was able to estimate the strength of the atomic bomb detonated at the Trinity test, based on the distance travelled by pieces of paper dropped from his hand during the blast.

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What is a Fermi Estimation?

A ‘Fermi’ estimation is simply the real-life term given for estimation. Don’t panic!

The aim of Fermi estimation is to provide a realistic estimate to a numerical problem by making a series of approximations and then using the four operations (+, -, ×, ÷) to reach an estimate.

This is by no means an accurate method of obtaining an answer but by following the process a ‘good’ estimate can be found with little working out, whereas an accurate answer may need far more detailed calculations or may not even be obtainable at all. It enables us to make ‘plausible estimates’ and it provides a way of checking to see if an accurate answer is correct.

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How do we estimate?

1. Round the numbers to 1 significant figure (see next slide)

2. Perform the calculation

3. Check your final answer is a plausible/realistic estimate

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Rounding (to 1 significant figure) – a quick recap

Rounding to 1 significant covers all types of rounding, depending on the size of the number you're being asked to round. It could require you to round the nearest … 1000, 100, 10, whole number, or a given number of decimal places.

To round to 1 significant figure:

1. look at the first non-zero digit if rounding to one significant figure
2. draw a vertical line after the place value digit that is required
3. look at the next digit
4. if it's 5 or more, increase the previous digit by one
5. if it's 4 or less, keep the previous digit the same
6. fill any spaces to the right of the line with zeros, stopping at the decimal point if there is one

EXAMPLES – Round to 1 significant figure

5389 → 5|**3**89 → 5000 689 → 6|**8**9 → 700 25,012 → 2|**5**.012 → 30,000

82 → 8|**2** → 80 14800 → 1|**4**800 → 10000 0.48 → 0.4|**8** → 0.5

If you need more help, watch the link below up until 2min20 seconds
https://corbettmaths.com/2013/09/07/rounding-significant-figures/

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Rounding (to 1 significant figure) – Your turn!

Round each of the following numbers to 1 significant figure:

Number	To 1 s.f.
4213	
87	
6435	
59,600	
4,325	
781	
23.6	
43.89	
0.0465	
0.009231	
0.9649	

If you need more help, watch the link below up until 2min20 seconds
https://corbettmaths.com/2013/09/07/rounding-significant-figures/

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Estimating – a quick recap

When finding an estimate or approximate solution, you are looking for a 'rough' answer. To get a plausible or realistic answer, we round each value to 1 significant figure first.

EXAMPLES – Estimating answers

Question	Rounded Question	Estimated Answer
312 + 487	300 + 500	800
884 – 623	900 - 600	300
47 x 8	50 x 8	400
558 ÷ 9.8	600 ÷ 10	60
$\frac{695 + 472}{6.2}$	$\frac{700 + 500}{6} = \frac{1200}{6}$	200

If you need more help, watch the videos below:
https://www.drfrostmaths.com/videos.php?skid=48
https://corbettmaths.com/2012/08/21/approximation-to-calculations/

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Estimating – Your turn!

Question	Rounded Question	Estimated Answer
1456 + 479		
1022 - 782		
9.8 x 83		
97 ÷ 22		
2679 + 2529		
3168 - 1726		
68 x 923		
$\frac{220 \times 3.5}{83}$		
£28.75 x 7.88		
$\frac{£22 \times 45}{2}$		

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Estimating – Your turn! ANSWERS

Question	Rounded Question	Estimated Answer
1456 + 479	1000 + 500	1500
1022 - 782	1000 - 800	200
9.8 x 83	10 x 80	800
97 ÷ 22	100 ÷ 20	5
2679 + 2529	3000 + 2000	5000
3168 - 1726	3000 - 2000	1000
68 x 923	70 x 900	63000
$\frac{220 \times 3.5}{83}$	$\frac{200 \times 4}{80}$	10
£28.75 x 7.88	30 x 8	£240
$\frac{£22 \times 45}{2}$	$\frac{20 \times 50}{2}$	£500

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Example of a Fermi Estimation

Problem: Find out the number of piano tuners in the city of Norwich.

First of all, an accurate answer can be obtained (by looking in business directories etc.) but we wish to get a 'plausible' estimate by making a series of approximations.

The estimates we need to make are:

- What is the population of Norwich?
- How many houses in Norwich?
- How many houses have pianos?
- How many times does a piano have to be tuned?
- How many pianos can be tuned per day?
- How many days does a piano tuner work?

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Example continued…

To find the size of the city of Norwich, do an internet engine search (not guaranteed to be correct!) and then round it to an 'easier' number. The exact population of Norwich is 214 000 so we approximate this to 200 000, to make the calculations easier.

The other assumptions…

- There are 4 people living in each house ('plausible') and so there will be 200 000 ÷ 4 = 50 000 houses in Norwich.
- Now how many pianos are in these houses? Let's make an assumption there is one piano every 50 houses, this means that there are 50 000 ÷ 50 = 1000 pianos in Norwich.
- Estimate how many times a year they need to be tuned; let's say twice a year.
  That means that there are 1000 × 2 = 2000 piano tunings a year.
- How many days does a tuner work? Let's assume they have the standard 4 weeks holiday and work for 48 weeks a year, not including weekends. To make the calculation easier, round to 1 s.f. first and hence work for approximately 50 × 5 = 250 days a year.
- If a tuner can do 2 tunings a day then there are 2000 ÷ 2 = 1000 days of work available to the piano tuners.
- Therefore as each piano tuner works for 240 days then the number of piano tuners is 1000 ÷ 250 = 4, so approximately **4 piano tuners.**

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Example – summary!

The assumptions made were:

- 200 000 people live in Norwich. (yes, this was an underestimate!)
- 4 people per household.
- 1 piano every 50 houses.
- 2 piano tunings a year.
- It takes 1 a day to tune a piano.
- A piano tuner works for 50 weeks of the year (yes, this was an overestimate!)

These assumptions are simply estimations of what happens in reality; but they are plausible estimates; they are not completely off the mark. You could argue that an estimate of 4 people per house is too much but if it is only 2 people per house, the size of the eventual answer doesn't change much. In fact it is this aspect of Fermi estimation which makes it so appealing. If we over-estimate or under-estimate the assumptions, they will balance themselves out in the overall answer!

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Challenge Time!

There are 5 Fermi Estimation Questions on the next slide.

We would like you to share your fully worked solutions with your Maths teacher by emailing them to your Maths Teacher by Friday 3rd July.
You can answer one, several or all of the questions.

As there is no 'correct' answer, your work will be judged based upon whether:

- you have considered all factors impacting on the estimate?
- you have clearly identified any assumptions you've made?
- you have done research to make assumptions reasonable?
- you have rounded correctly?
- you have performed accurate calculations to arrive at your estimate?
- your final answer a 'plausible estimate'?

Bonus points awarded if it's close to the estimates found by the Maths department!

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Fermi Estimation Challenge!

1. How many ping pong balls would it take to fill a classroom?

2. How many metres of spaghetti does the school need to buy if Spaghetti Bolognese is on the menu?

3. How many miles are walked/run by the everybody in the UK during a week under current lockdown conditions?

4. Assuming social distancing rules are followed, how many customers could McColl's shop in Tupton serve in one day?

5. What's the maximum number of people that could fit on the school site and how would this change if they all socially distanced responsibly?

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